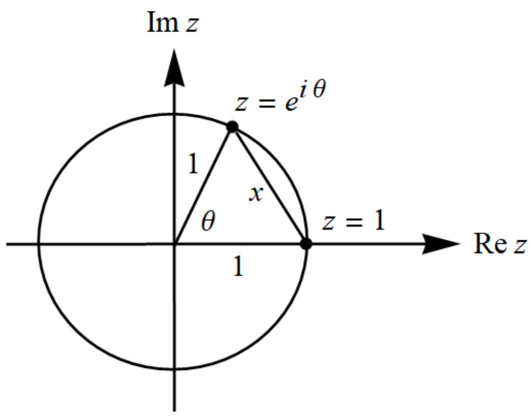
Exercise 4

Using the fact that the modulus $|e^{i\theta} - 1|$ is the distance between the points $e^{i\theta}$ and 1 (see Sec. 4), give a geometric argument to find a value of θ in the interval $0 \le \theta < 2\pi$ that satisfies the equation $|e^{i\theta} - 1| = 2$.

Ans. π.

Solution



Use the law of cosines.

$$\begin{aligned} x^2 &= 1^2 + 1^2 - 2(1)(1)\cos\theta \\ &= 2 - 2\cos\theta \end{aligned}$$

Set x, the distance between $z = e^{i\theta}$ and $z = 1$, to 2 and solve for θ.

$$2^2 = 2 - 2\cos\theta$$

$$2 = -2\cos\theta$$

$$\cos\theta = -1$$

$$\theta = \pi + 2\pi n, \quad n = 0, \pm 1, \pm 2, \ldots$$

Since we require $0 \le \theta < 2\pi$, we choose $n = 0$. Therefore,

$$\theta = \pi.$$